Minor-1: ELL212/EEL207 Engineering Electromagnetics, 2015-16 Semester 2

Instructions: NO mobile phones on person; one sided A4 cheat sheet allowed; read the questions very carefully, and show all intermediate steps. Total points : [22]. Code: V5Y
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Basics [No more than one/two line answers; anything more will not be evaluated]

1. Show that mangetic forces do no work. [1]
2. State the static boundary conditions for the electric field at the interface between air and a (charged) perfect conductor. [2]
3. Write down Gauss's and Ampere's law in terms of free charges and currents only. [1]
4. If I have a computer code that solves Poisson's equation, can I use the same code to solve for the magnetic vector potential given the current distribution? Yes/No/How. [1]
5. The usual form of Ohm's law is $\vec{J} = \sigma\vec{E}$. What will it be if magnetic forces are also to be considered? Write the expression and give a one line explanation. [1]

Beyond basics [Be sure to reason out all intermediate steps; correct answers without reasoning don't count. Do NOT rewrite the question]

6. The lossless line shown in Fig. 1 is operating with $\lambda = 100$ cm and $Z_0 = 75\Omega$. If $d_1 = 10$ cm, $d = 25$ cm, and the line is matched to the left of the stub, what is Z_L? [3]

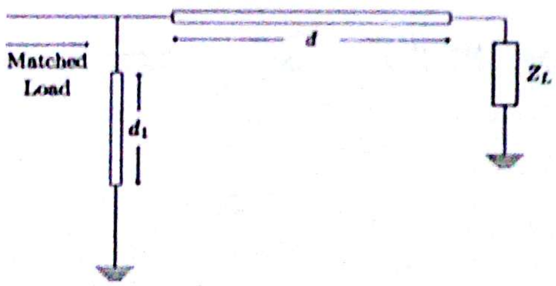

Figure 1: Find the load, Z_L.

7. Recall the experiment with the frog that was made to levitate. Imagine you have access to a device which can produce a magnetic field of the form $\vec{B}(z) = B_0\sqrt{a+bz}\,\hat{z}$, where z is the vertical coordinate in the lab (all experiments are conducted in the region above where $a + bz > 0$, and a, b are constants). What should the sign of b be in order for the frog to levitate? Obtain an approximate expression of the field strength required for the experiment to succeed. *Hint:* The force on a magnetic dipole in a field is $\vec{F} = \nabla(\vec{m}\cdot\vec{B})$. Make reasonable assumptions and state them. [4]

8. You are on your way back from ELL212 class when you get abducted by an alien ship which takes you to their home planet, which, turns out to be very similar to ours, except that they have only two spatial dimensions! They know that you know a thing or two about electrostatics (and they don't) and ask your help in framing their laws. After your own form has been converted to two dimensions, you make some measurements and find out that as the separation between charges is doubled, the force between the charges get reduced by a factor of 2. What must be the simplest form of Coulomb's law (is the force conservative?), the electrostatic potential, and the equivalent to Gauss's law? [4]

9. Having succeeded in satisfying the 2D aliens above, they ask for you help in resolving certain wave phenomena in their two-dimensional world. Assuming a point-source for a wave, they ask you to write a solution for the wave equation. What is this general solution? Your solution will contain certain unknown constants; how will you determine them?
Use your knowledge of the fact that the Bessel differential equation, given as: $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2 - n^2)f = 0$, $n \in \mathbb{Z}$ has solutions that can be written as a linear combination of two linearly independent functions, $J_n(x)$ and $Y_n(x)$. *Hint*: Consider polar coordinates. What might be the θ dependence of a wave from a point-source? [5]

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IMPORTANT: Write your question paper code on your answer sheet.

Figure 2: (Credit: Wikipedia) Some relations in vector calculus. 2D polar equations can be derived by removing the z coordinate from the cylindrical relations

Operation	Cylindrical coordinates (ρ, φ, z)	Spherical coordinates (r, θ, φ)
A vector field $\mathbf{A}$	$A_\rho\hat{\boldsymbol\rho} + A_\varphi\hat{\boldsymbol\varphi} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\theta\hat{\boldsymbol\theta} + A_\varphi\hat{\boldsymbol\varphi}$
Gradient ∇f	$\frac{\partial f}{\partial\rho}\hat{\boldsymbol\rho} + \frac{1}{\rho}\frac{\partial f}{\partial\varphi}\hat{\boldsymbol\varphi} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\boldsymbol\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\hat{\boldsymbol\varphi}$
Divergence $\nabla\cdot\mathbf{A}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$
Curl $\nabla\times\mathbf{A}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\hat{\boldsymbol\varphi} + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\varphi}\right)\hat{\mathbf{z}}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol\varphi}$
Laplace operator $\nabla^2 f \equiv \Delta f$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$

$dl = dR,\ R\,d\theta,\ R\sin\theta\, d\phi$